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ABSTRACT

This paper focuses on student understanding and learning of the algebraic concept--rate of change. Teachers' knowledge about their students and current literature on students' understanding is used to design and assess the instruments. Data were obtained as part of two-year research program that focused on how the study of students' understanding of functions and algebra affected multiple systems in an urban bilingual high school. The analysis of data by class showed some information about the relationship between classroom curriculum and outcomes for the class. (ASK)

STUDENT'S UNDERSTANDING OF RATE OF CHANGE: THE USE OF DIFFERENT REPRESENTATION

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This paper focuses on "student understanding and learning" of an algebraic concept – rate of change. We have begun to create maps of the students' development in this particular content area. Students' development is constrained by the school conditions, classroom environment, and teachers' knowledge and beliefs (see Secada, 1999, Theoretical framework). These constraints have been taken into consideration in the design on this part of the project. We use teachers' knowledge about their students and current literature about students' understanding to design and assess the instruments presented in this paper.

Understanding is characterized by the ability to see how things are related or connected to other things we know. Understanding is not static, it is always changing and growing (Hiebert, et al., 1997). Students' understanding about algebra, in particular rate of change, and teachers' understandings about students' thinking were the focuses of this project. We hypothesized that in order to teach for understanding teachers would need to understand their own students' thinking. Having students' thinking as the focus of teachers conversations, gave teachers opportunities to reflect about their practice, to learn about students' algebraic thinking, to use this knowledge to plan instruction, and to have a common focus for mutual engagement with other teachers in professional communities.

Carpenter and Lehrer (in press) propose five mental activities to help develop mathematical understanding: "(1) constructing relationships, (2) extending and applying mathematical knowledge, (3) reflecting about experiences, (4) articulating what one knows, and (5) making mathematical knowledge one's own" (p. 2). We used three of these activities as underlying principles in constructing a paper-and-pencil test to get at students' understanding of rate of change and linear models. The use of different representations to study functions, in particular linear functions, allowed us to look for relationships between situations presented using graphs, tables, symbols, or natural language. The tasks were designed to look for how students extend and apply knowledge they had seen before in the classroom. The tasks were similar to the ones the teachers had given the students, but included some extensions, which allowed students to show different levels of understanding. Students were also asked to explain their reasoning. By articulating their thinking we were able to trace different learning trajectories.

The concepts of functions and modeling are central for conceptualizing algebra (Kaput, 1995). He stated that instead of a set of rules for transforming symbolic representation into equivalent but simpler expressions, algebra is now considered as a way of thinking that involves representing and modeling quantitative information, generalizing patterns and structures, and

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representing relationships using variables and functions. The tasks used to tap students' thinking, included interpretation and modeling of situations using different systems of representation, generalizing patterns, and the use of symbolic representation to describe relationships.

METHODS

The data discussed here were obtained as part of a 2-year research program, which focused on how the study of students' understanding of functions and algebra affected multiple systems in an urban bilingual (Spanish-English) high school. Fieldnotes, classroom observations (audiotaped), and teacher interviews (audiotaped) and a paper-and-pencil test were used as data collection tools. Analysis for this paper focuses on the paper-and-pencil test.

Participants

Sixty-three algebra students, from three different bilingual classes, took the paper-and-pencil test about rate of change. Sixty percent of the students were female and forty percent were male. The students came from two different algebra courses: 66% from the Algebra I and 34% from the Algebra II.

The Algebra I course at this high school focused on linear functions, or in other words, finding patterns, writing equations, graphing lines, and finding the best fitting line for a set of data. Other topics included solving equations, systems of equations, and inequalities. The Algebra II course reviewed and extended the content of Algebra I. Linear, quadratic, and trigonometric functions were the focus of the course. In this study, two Algebra I and one Algebra II classes were studied.

Table 1
Teachers classes and student breakdown

Teacher	Course	Students		
		Female	Male	Total
Isabel	Algebra I	14	11	21
Gustavo	Algebra I	10	7	17
Fernanda	Algebra II	14	7	21

Isabel taught one of the Algebra I classes. There were 25 students in her class, 14 females and 11 males. Problems related to rate of change were common in her class even though she never used the term "rate of change" in connection to these problems. Isabel commonly gave problems where the emphasis was on finding patterns by using tables and determining the corresponding equation, and then answering some questions related with the original situation. The use of graphs in this type of problem was supported by graphic calculators. There was no evidence that students had opportunities to translate from graphs to tables or from graphs to equations.

Gustavo taught the other Algebra I class. There were 17 students in his class, 10 females and 7 males. All the students in this class had failed Algebra I previously. Rate of change was a

specific topic for Gustavo, connecting the unit of proportion to linear equations and exploring the concept of slope in tables, graphs, and natural-language. A typical practice in Gustavo's class was to give a problem where he had students either explore the pattern in a table setting or information from a graph. In both cases students had to write equations describing the situation and then use the equation to solve the problem. The terms "rate of change" and "slope" were commonly used as well as the notation $\Delta y/\Delta x$.

Fernanda taught Algebra II. There were 21 students in her class, 14 females and 7 males. All of Fernanda's students were juniors and seniors. Fernanda viewed problems involving rate of change as a review topic. Therefore, there was no evidence of exploring patterns in a table or reading information from a graph related to rate of change. She emphasized symbolic manipulation of linear functions and inequalities. Because the student population in Fernanda's class consisted only of juniors and seniors students, she spent a considerable amount of time preparing them for the proficiency examination, which was a graduation requirement at this school. Rate of change is one of the topics on this examination. Consequently, problems involving rate of change were explored in Fernanda's class along with the other topics and were presented as open-ended situations where it was up to the student to find his or her own way to solve the problem.

Procedures

The paper-and-pencil test was administrated by the teachers in their classrooms at the end of the 1997-1998 school year. Students were given a one-hour class, 55 minutes, to complete the test. Each question in the test was presented in English and Spanish. The test consisted of three questions, each one constructed to bring into focus one of the representations of rate of change — table, graph, or natural language (see Appendix A for the actual test).

The test was scored using a rubric designed to take into account different levels of understanding evidenced from student's responses. First, responses were scored as right or wrong secondly, different methods or strategies were classified. In overall, the rubric was used to assign low scores to answers showing low levels of understanding and high scores to answers showing deeper (see Appendix B for a detailed description of the rubric). No attempt to answer the questions and answers that did not fit any of the other categories were coded separately. Two members of the research team scored each test separately. When differences in scoring arose, they discussed it until agreement was reached.

RESULTS

Table problem

The first problem presents a situation in a table format (see Appendix1). MacGregor and Stacey (1995) found that students work more easily finding a pattern down one column of a table than trying to find a functional relation across columns of tables. In terms of rate of change, students without a good understanding of rate of change may focus on one variable at a time rather than the covariation of variables.

The table in this problem includes a schedule of payments over a period of time. Students were asked to explain the rate of change corresponding to the data in the table and to find different missing values in the table. The last parts of the questions required the use of rate of change to find new values related with the situation. About 24% of the students did not solve this problem or gave completely wrong answers.

In problem 1 students were to recognize the pattern and give qualitative and quantitative descriptions of the rate of change involved in the problem. The qualitative answers focused on characteristics of the situation (e.g. as time went by the daughter owed her mom less money). Quantitative answers related the numbers and the relationship between those numbers (e.g. every 10 months the daughter pays $150).

When the problem asked for completing the values of the table, most students tended to solve the problem by looking at change in only one of the variables. These students did not recognize the correspondence or covariation between the two variables involved in the problem (time and money). Common answers involved an additive strategy to find the value of x or y, following the pattern in the previous values of the correspondent column. For example, a common answer for y was $200 (because in foregoing cells the amount owed changed by 150, the previous value was $350). Students here did not consider the change in time (see Figure 1).

y=200
Since the rate is going down by 150
then I substracted 350-150=200.

Figure 1. Common wrong answer for 1b.

The few students who were able to recognize the correspondence between the variables, used strategies similar to the ones used in solving missing-value proportional problems (see Figure 2). In this case they established a proportional relationship between the two variables in the problem.

35 •	y 275
50 x	50

Encontre la respuesta basandome a que si cada
diez meses se da la cantidad de $150 ahora
van hacer cinco meses yo podria restar entre
cinco meses más para poder encontrar el valor que
fue de $75 cada cinco meses.

Translation: I got my answer realizing that every ten months the amount is $150 and now is changing to five months. I could take out five more months to find out the value which was $75 for each five months.

Figure2. Correct answer for 1b

Seventeen percent of the students showed evidence of covariational understanding to find a relation between the variables. These students found the rate of change of the amount paid monthly and used that to solve correctly for the missing value (see Figure 3). Even though they did not use formal algebraic equations, their answers showed clear evidence of understanding of the relationship between time and money.

$$\frac{10 \text{ meses}}{\$150} = \frac{x \text{ meses}}{\$750}$$

$$\frac{150x}{150} = \frac{\$7500}{\$150}$$

$$x = 50 \text{ meses}$$

Figure 3. Correct answer for 2c.

Graph problem

With respect to graphs, researchers have found that students have some problems determining rate of change, such as, confusing the meaning of the slope with the meaning of the height of a graph, looking at graphs as pictures of a situation, and dealing better with graphs pointwise than across-time (Clement, 1985; McDermott, Rosenquist, & vanZee, 1987; Monk, 1992).

Our second problem presented linear graphs of a 100-meter race run by a mother and her daughter. The mother started 20 meter ahead. Students must answer the first two questions based on a graph without numbers on the axis. These questions ask who won the race and who was the faster runner. The next four questions require the analysis of the rate of change of the

distance in relation to time for each of the runners and of the equations which relate distance and time for each runner. Twenty four percent of the students did not solve this problem or gave completely wrong answers.

About 48% of the students gave a qualitative description of the information based on the graph. In other words, these students were able to read the necessary information from the graph and identify the winner of the race and who ran faster. Students had difficulty with this situation. Few students were unable to follow the lines or mixed them in their analysis (e.g. they lost track of the lines after the intersect point). Another common mistake was the interpretation of the graph as distance vs. speed instead of distance vs. time. These results were not surprising; Monk (1992) had described similar student mistakes.

About 27% of the students calculated the rate of change correctly when the y-intercept was 0. When the y-intercept differed from zero, 21% of the students found the rate of change correctly (see Figure 4). The most typical answer involved using the end-points of the lines, ignoring the y-intercept and identifying the rate of change with the relation (quotient) between the coordinates of the end-point. Some students used points chosen randomly which showed understanding of the relation between the variables. Other students identified "nicely situated" points, described the relation between the coordinates, and wrote the rate of change (see Figure 5).

She ran 100 meters in 18 seconds.
So for every second she ran 10 meters.

Figure 4. Answer for 2c ignoring the y-intercept.

Mother, $\frac{\Delta y}{\Delta x} = \frac{20-60}{0-9} = \frac{40}{9}$

I get a point for meter 20 and 60, than
I went to second to get the other points
were they could contact my points of
20 and 60 than I did a rate of change
and subtract and gave me the answer $\boxed{\frac{40 \text{ meters}}{9 \text{ seconds}}}$

Figure 5. Answer for 2c identifying two nicely situated points.

Approximately 76% of the students did not attempt writing equations that related relationships between time and distance. Twenty-one percent attempted, but only 3% of these students were able to write correct equations for the two runners.

Natural language problem

We also chose to study rate of change in a natural-language situation. One reason for this is we saw this type of representation often used in instruction. A second reason to use this task is that studies have shown students show a certain preference among types of representations of function. This enables us to see whether this is also true when solving rate of change problems (Dreyfus & Eisenberg, 1982; 1984). The third task was open allowing students to choose any representation—tabular, graphical, algebraic, etc.— to solve the problem.

The last problem presents two summer job offers. The hourly pay rate, the amount of time worked, and the cost of one uniform are given in the problem. Students were asked to choose the better-paying offer and explain their reasoning. Forty-one percent of the students did not solve this problem or gave completely wrong answers.

When the test was designed we assumed students would use different representations to solve it. Looking through their answers, we were surprised by the use of informal methods to make sense of this problem. When analyzed more carefully we realized that the use of any formal method to answer the questions was not necessary. It made us change the way of classifying students' responses. Instead of looking for different representations, we analyzed the type of arguments used. Were students using qualitative or quantitative arguments? Were they basing their argument on one week, twelve weeks, or another time period? What personal knowledge about the situation were students using to make sense of the problem?

Thirty-seven percent of the students solved this problem correctly. Out of this group, most gave a calculation response for the first week and a qualitative explanation for what would happen over the summer (see Figure 6). Many students who did not solve the problem correctly used calculations for the first week to support their conclusion. Eight percent of the students used calculations for more than one week to argue for the better option.

1) $4.50 → pagan la hora
20x horas para trabajar
$90.00 → por semana lo gana
-45.00 → uniforme
$45.00 → lo que gano la 1ra semana

2) $3.50 → pagan la hora
20x → horas para trabajar
$70.00 por semana lo que ganó

En la oferta 1 pagan mejor dinero, porque en la oferta 2 le pagan $20.00 menos que son $70.00 por semana. En la 1 le pagan $90.00 por semana. Lo unico que en la oferta 1 esque en la primera semana va pagar $45.00 por el uniforme, y el pago por la 1ra semana va a ser $45.00 porque le van a rebajar el pago del uniforme. Pero después ya seguir ganando $90 00 por semana. Y en la 2 va seguir ganando $70.00. Yo pienso y creo que la oferta #1 daría el mejor pago para trabajar durante el verano.

Translation: Offer 1 pays more money because Offer 2 pays $20 less which is $70 per week, Offer 1 pays $90 per week.In the first week Offer 1 pays $45 because of the uniform and you get for the first week $45 because they take away the money for the uniform. But after that you get $90 per week. And in Offer 2 you'll keep getting $70. I think and believe that Offer 1 would give you better payment for your work during the summer.

Figure 6. Answer for 3 using calculations for week 1 and qualitative analysis for the long term.

In this problem we found a lot of context variations. Students included their personal enlightenment based on their own knowledge and experience in similar situations. One example of "context knowledge" was using what they knew of real experiences to buy more than one uniform for the summer job. Other contextual knowledge included determining the nicer place to work, a food place rather than a car wash. Some even stated that you do less work in a food place than in a car wash.

Summary

The analysis of the data by class showed some information about the relationship between classroom curriculum (as content covered) and outcomes for the class. It also informed

us about the flexibility students have in their understanding, or in other words, whether and how students' reasoning in one situation is carried over to another situation.

In general, Isabel's students scored better than students did in other courses for the table problem. More specifically, Isabel's students did better on this problem than on the other problems. In fact, they did better on the graph problem than on the natural language problem. This result shows how emphasizing one type of representation, in this case tabular, affects how students work within one predominant representation.

Gustavo's students did better than students in the other classes on the graph problem and they did better on this problem than on the other two. His students had more exposure to graphs and natural language situations in which it was necessary to explore graphs. This may have contributed to his students' success in developing strategies to solve this type of problem.

Fernanda's students did not score better than students did in the other classes on any problem. In the table situation they scored the same score as Gustavo's students. For the graph problem they scored slightly better than Isabel's students. In the natural language problem students from all three classes scored the same. This result is important since shows these students have no retention from the topic studied in Algebra I. It shows also that reviewing the topic did not give them tools to solve problems that require conceptual understanding of rate of change.

DISCUSSION

When analyzing students' responses we were able to create levels of understanding for each problem situation. The definition of these levels was our attempt to create a learning trajectory of students' understanding of linear models. The remaining years of the project will provide us with further information to complete this trajectory and help determine the link between different representations. Table 1 presents the distribution of students in each level and for each situation and Table 2 defines the levels.

Table 1.
Percentage of students in each question and in each level

Table Situation (n=61)		Graphic Situation (n=63)		Natural Language Situation (n=61)	
Level	Students	Level	Students	Level	Students
I	28%	I	25%	I	43%
II	39%	II	32%	II	16%
III	15%	III	19%	III	1%
IV	10%	IV	8%	IV	30%
V	8%	V	10%	V	8%
		VI	6%	VI	0%

Within the tabular representation there were five levels of understanding identified. Level I represented no evidence of understanding. In Level II a student showed very limited understanding of rate of change giving poor qualitative and/or quantitative descriptions that looked at only one variable. At this level there was no evidence of understanding of covariance. In Level III, qualitative reasoning about rate of change was still poor, but their quantitative descriptions improved. That is, a student at this level was able to see and explain the pattern in the table, but was not able to use that information to answer the other questions. Level IV included responses in which students were able to give both qualitative and quantitative descriptions of rate of change. At this level, understanding of covariation was well developed, but they were not always able to use the covariation to answer further questions. Finally, in Level V, students were able to use their understanding of covariance to answer the extended questions (See Table 2).

Within the graphical representation there were six levels of understanding identified. Level I represented no evidence of understanding. Level II portrayed students who gave a qualitative description of the information presented in the graph, but were not able to quantify or generalize that information. In Level III, students gave qualitative descriptions and attempted to quantify the information from the graph to figure out the rate of change. They were not, however, able to generalize or give an equation that represented the given graph. At Level IV students gave qualitative descriptions and found the rate of change when the y-intercept was equal to zero. However, they did not extend that knowledge to a situation when the y-intercept was not equal to zero. Similar to level III, students did not generalize the information from the graph or write an equation. Students at Level V were, in addition to what has already been described, able to find the rate of change when the y-intercept was not equal to zero. There was also an attempt to generalize and write an equation. At Level VI, students were able to take the last step, generalize from the situation and write an equation (See Table 2).

For the natural language problem there were six levels of understanding identified. Again, Level I represented no evidence of understanding. Level II represented students' responses that gave some qualitative explanation to figure out the calculation for one week even if the calculations are wrong. Students at Level III gave correct calculations for the first week, but limiting the analysis to one week gave them the wrong answer. At Level IV students calculated correctly the first week's pay and used their qualitative understanding of the situation to anticipate what would happen as time goes by. By combining their quantitative facts and their qualitative understanding they generated the correct answer. Students at Level V gave correct calculations for more than one week and used this information to generate a correct answer. Level VI illustrates understanding of algebraic manipulation. These students used equations, tables, or graphs to show their analysis of the situation. It is important to mention that we believe that this problem did not necessarily offer the students to use their most sophisticated strategies. Therefore, it is difficult to say that students that show Level IV of understanding are not able to show understanding corresponding to Level V or even Level VI understanding if the problem forces the use of those sorts of strategies to generate an answer (See Table 2).

Table 2.
Levels of understanding in each problem

	Problem 1 Table situation		Problem 2 Graphic situation		Problem 3 Natural language situation
I	No attempt to answer or completely wrong answer	I	No attempt to answer or completely wrong answer	I	No attempt to answer or completely wrong answer
II	Attempt to answer all questions, poor qualitative and quantitative description of rate of change, no co-variation sense, not able to use it	II	Attempt to give qualitative description of the information presented in the graph, not able to describe the rate of change or to generalize and write equations	II	Able to produce some kind of explanation or calculations for the first week, use that information to generate their answer, wrong answer
III	Poor qualitative descriptions, correct quantitative description, no sense of co-variation, not always able to use it	III	Able to give a qualitative description of the information presented in the graph, attempt to give quantitative descriptions of rate of change, not able to generalize and write equations	III	Able to do some calculations to get quantitative information for more then one week, but not able to apply that to generate the right answer
IV	Able to give a qualitative or quantitative description of rate of change, see co-variation, not always able to use it	IV	Able to give qualitative descriptions of the graph, able to find the rate of change when y-intercept = 0, cannot extend that when y-intercept $\neq 0$, not able to generalize and write equations	IV	Able to use calculations to correctly to get quantitative information for one week, but add a qualitative descriptions of what will happen eventually and generate right answer
V	Able to give a qualitative and quantitative description of rate of change, see the co-variation of the variables, use this co-variation to find a new information	V	Able to give qualitative descriptions of the graph, able to find the rate of change independent of the y-intercept, attempt to generalize and write equations	V	Able to use calculations correctly to get quantitative information for more then one week, and use that information to generate their answer
		VI	Able to give qualitative description of rate of change, description of the relations between time and distance included, able to find the rate of change independent of the y-intercept, able to generalize and write equations	VI	Able to use algebraic manipulation in their solution strategy, that is equations, table or graphs

CONCLUSION

The data presented here has provided us with initial information about how students understand rate of change and linear models. This information enables us to better understand students' thinking. Continuing our work will allow a more detailed picture of students' understanding of rate of change.

The hypothetical learning trajectory (Simon, 1995) is a model that enables teachers to hypothesize a developmental path the students might take in their learning process. In order to anticipate such learning path teachers must understand students' reasoning about a specific concept. The data has provided us, the research team and the teachers, with knowledge of how students come to understand rate of change and linear modeling and, consequently, their learning trajectory. With this knowledge teachers will be more qualified to interpret patterns in their students' thinking and be responsive to their understanding in the classroom. Also, this knowledge will help teachers construct a sequence of meaningful tasks that might highlight the development of students' thinking. Furthermore, these tasks allow students to follow a sense-making process, which extends their understanding of the domain.

The base system, which entails student's understanding of specific content area, is on which the other two systems rest on. During this year we have conducted workshops for teachers, presenting them with this information of students' reasoning and how to focus on students' way of thinking in the classroom practice. Such knowledge will help the teachers design a learning environment that will promote student understanding. Our continuous work will enable us look at how teachers' knowledge about their students' understanding of rate of change and linear models will perturb the other two systems, that is their knowledge and beliefs and classroom environment. We hope that this knowledge about students' thinking will drive changes in the algebra curriculum, making more emphasis on connections between representations, generalization, and real situations in which algebraic concepts have application. We believe that a better knowledge about students' understanding will lead the teachers to confront issues about curriculum, instructions, and assessments.

In the next step of this research, changes suggested by the teachers, and some others due to the results in the first implementation, will be made, and the test will be given to a new group of students. The purpose will be to get a more refine definition of the levels presented above and to pose some hypotheses related with the relationships among the use of different representations.

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APPENDIX A
Paper and Pencil Test

Students Test

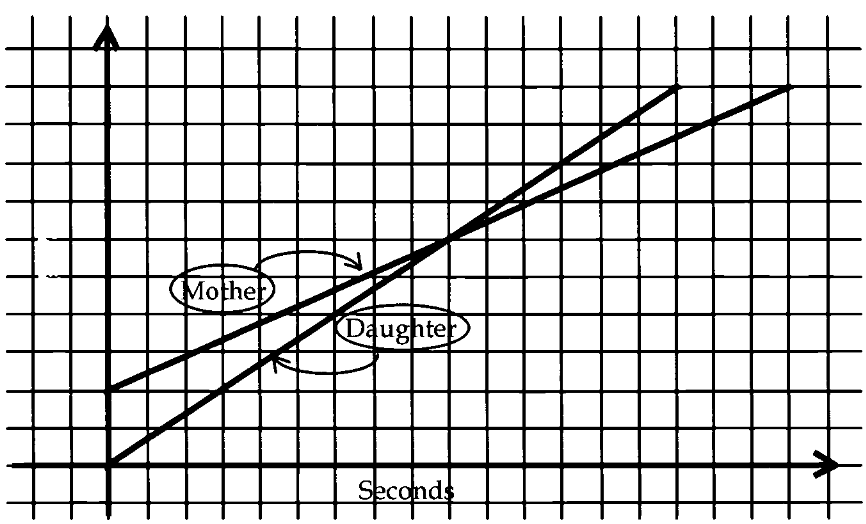
Problem No. 1

Juana wants to go to a summer camp that cost $800. Juana asks her mother for a no-interest loan in order to pay for the summer camp. Her mom sets a schedule of payments that looks like this:

Number of months	Amount still owed
0	800
10	650
20	500
30	350
35	y
x	50

a. Describe the rate of change in words.
b. Find the value of y in the table. Show or explain how you got your answer.
c. Find the value of x in the table. Show or explain how you got your answer.
d. How much does Juana pay her mother each month? How do you know?
e. How much did Juana still owe her mother after 21 months? Show or explain how you got your answer.

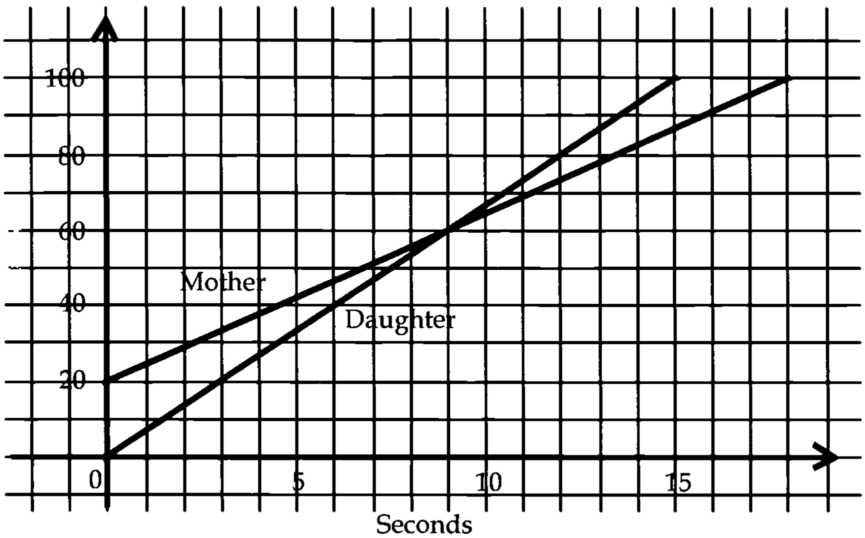
Problem No. 2
The following graphs represent the progress of a 100-meter race run by a mother against her daughter.



The graphs give the distance from the starting line of each contestant t-seconds after the start. Answer the following questions from the information given by the graph.

a. Who won the race? Explain how you know.
b. Who ran faster during the race? Explain how you know.

Use the graph below to answer the next questions about the race.



c. Calculate the rate of change (distance in relation to time) for the mother. In other words, how many meters per second did the mother run? Show how you got your answer.
d. Calculate the rate of change (distance in relation to time) for the daughter. In other words, how many meters per second did the daughter run? Show how you got your answer
e. Write an equation to express the relationship between distance and time for the mother. Show how you got your answer.
f. Write an equation to express the relationship between distance and time for the daughter. Show how you got your answer.

Problem No. 3
You have received two job offers for the summer. Determine which one would be the better-paying summer job. Explain all your reasoning and show your work.

Offer 1:
At Tico's Tacos you will earn $4.50 an hour. However, you have to purchase a uniform for $45.00. You have to work 20 hours each week.

Offer 2:
At Carla's Car Wash you will earn $3.50 an hour. No special uniform is required. You have to work 20 hours each week.

APPENIX B
Scoring Rubric

Problem	Question	Score	Description
1	a	-99	No answer
		0	Wrong
		1	Description of change on one variable (time OR money)
		2	Qualitative description of rate of change
		3	Quantify description of the rate of change
		188	Non scorable
	b	-99	No answer
		0	Wrong
		1	Description of change in one variable (money) (200)
		2	Correct answer -no explanation (275)
		3	Correct answer with explanation (275)
		188	Non scorable
	c	-99	No answer
		0	Wrong
		1	Description of change in one variable (time) (40)
		2	Use of the change in money (150 dollars every 10 months) to explain a change in time of 10 months (45)
		3	Correct answer- no explanation (50)
		4	Correct answer with explanation (50)
		188	Non scorable
	d	-99	No answer
		0	Wrong
		1	Looking just at change in money (150)
		2	Correct answer- no explanation (15)
		3	Correct answer with explanation (15)
		188	Non scorable
	e	-99	No answer
		0	Wrong
		1	Show the rate of change per month is 15 but instead of subtract, add 15 to 500 (515) OR multiply 15 times 21 (315)
		2	Qualitative description of the relation between time and money(485)
		3	Quantify (equation) description of the relation between time and money (485)
		188	Non scorable
2	a	-99	No answer
		0	Wrong
		1	Wrong answer, explanation using the graph (mother's line is above of or is longer than daughter's, the mother starts ahead)
		2	Correct answer no explanation or incomplete (daughter)
		3	Correct answer, explanation relating the same distance in less time (daughter)
		188	Non scorable
	b	-99	No answer
		0	Wrong
		1	Wrong answer (mom) because she stars above or she finishes before
		2	Correct answer, wrong explanation (the graph is going up)

		3	Correct answer, no explanation or incomplete (daughter)
		4	Correct answer, explanation relating time and distance for both (daughter)
		5	Correct answer, explanation using slope (daughter)
		188	Non scorable
	c	-99	No answer
		0	Wrong
		1	Wrong answer, don't consider the mother 20 meters head start
		2	Wrong answer, misreading of the graph (mother's line is the one on top)
		3	Wrong answer, correct procedure (use two approximate points, 21/5)
		4	Correct answer , no explanation (80/18 = 4.44)
		5	Correct answer with explanation (80/18 = 4.4)
		188	Non scorable
	d	-99	No answer
		0	Wrong
		1	Wrong answer, misreading of the graph (mother's line is the one on top)
		2	Incomplete answer, describing distance per time qualitative without a concrete reference to rate of change (100 m per 15 sec)
		3	Wrong answer, correct procedure (use two approximate points, 31/5)
		4	Correct answer, no explanation (100/15 = 6.66)
		5	Correct answer with explanation (100/15 = 6.66)
		188	Non scorable
	e	-99	No answer
		0	Wrong
		1	Wrong (relate time and distance but in a wrong way x+ y = 20 or x + y = 100)
		2	Wrong answer, instead of adding 20, subtract 20 (y= 4.4x-20)
		3	Correct answer using the wrong answer in c (y = 20 + ...x)
		4	Correct answer no explanation (y= 20 + 4.4x)
		5	Correct answer with explanation (y = 20 + 4.4x)
		188	Non scorable
	f	-99	No answer
		0	Wrong
		1	Wrong (relate time and distance but in a wrong way x+ y = 20 or x + y = 100)
		2	Correct answer using the wrong answer in d (y = ...x)
		3	Correct answer no explanation (y= 6.6x)
		4	correct answer with explanation (y = 6.6x)
		188	Non scorable
3	a	-99	No answer
		0	Wrong
		1	Qualitative explanation for the first week, wrong answer
		2	Correct calculation for the first week but wrong answer
		3	Correct calculation for more than one week, wrong answer
		4	Correct calculation for one week, qualitative explanation in long run, right answer
		5	Correct calculation for more than one week, correct answer
		6	Correct calculation, using equations and finding the intersection point, correct answer
		188	Non scorable

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